X special, glatt.
$\tilde q(X) = 0$

$$\mathrm{Alb}(\tilde X, o) \to \mathrm{Alb}(Y, E)$$
$$\downarrow$$
$$\mathrm{Alb}(Y, o) = 0$$

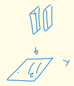
$$\begin{tikzcd} X \arrow[d, dashed, "\mathrm{MRC}"] & \tilde X \arrow[l] \arrow[d, "f"', "\mathrm{MRC}"] \\ Y \arrow[r, phantom, "="] & Y \end{tikzcd}$$
$\tilde X$ ← glatt.
Y ← glatt

$f : X \to Y \subset K3$

$$0 \to \underbrace{f^*\Omega^1_Y} \to \Omega^1_X \to \Omega^1_{X/Y}/\mathrm{tor} \to 0$$
$$\mathcal{F} \hookrightarrow \Omega^1_X, \quad \mathcal{F} \to 0$$

$\mathcal{E} \hookrightarrow \Omega^1_X$ $\quad \gamma : \hat X \to X$

$\mathcal{F}_i = \mathrm{Ind.}$ Ann. $H^0(\gamma^*\Omega^1_X)$



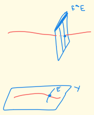
· $\mathrm{rg}\,\mathcal{F} = 1$. $\mathcal{F}|_C$ nef $\forall C$ allg.

$\mathcal{F} \subset \mathcal{F}^{sat}|_C \subset f^*\Omega^1_Y|_C =$ flach
$\Rightarrow \mathcal{F}^{sat} = \mathcal{O}_X$.

· $\mathrm{rg}\,\mathcal{F} = 2$:
$\mathcal{F}^{sat} = f^*\Omega^1_Y$

$$0 \to f^*\Omega^1_Y(D) \to \Omega^1_X \to \Omega^1_{X/Y}/\mathrm{tor} \to 0$$

$$\mathcal{F} \hookrightarrow f^*\Omega^1_Y(D)$$



$\sigma \in H^0(\hat X, \gamma^*\Omega^1_X)$

$$\prod_{g \in G} g^*\sigma \in \mathrm{Sym}^n \gamma^*\Omega^1_X = \gamma^* \mathrm{Sym}^n \Omega^1_X$$

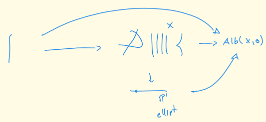
$$\begin{tikzcd} \hat X \arrow[d, "\gamma \text{ - } G\text{-galois}"] \\ X \end{tikzcd}$$

Prop $f : X \to Y$ Faserung proj.
mf. mit rat. zshgd. Fasern
$\Rightarrow$ orbifold-base von $(X, 0)$ ist $(Y, 0)$.
(Equiv: $\forall$ Primdivisoren $D \subset Y$
$\exists$ Primdivisor $E \subset X$:
. $f(E) = D$
. $\mathrm{mult}_E f^*D = 1$)

Ziel/Korollar nat. map
$$\mathrm{Alb}(X, 0) \to \mathrm{Alb}(Y, 0)$$
ist iso von $\mathbb{C}$-gruppen.

$$\begin{tikzcd} X \arrow[r] \arrow[d] & (\text{Torusquot.}, \text{bdry}) \\ Y \arrow[ur, dashed, "\exists!"'] \end{tikzcd}$$

Ziel 2 Y proj. mf, special,
nicht uniruled, $\tilde q(Y) = q(Y)$
$\Rightarrow \mathrm{Alb}(X, 0) = \mathrm{Alb}(Y)$.


X
$\to \mathrm{Alb}(X, 0)$
$\mathbb{P}^1$
ellipt.

$$\begin{tikzcd} X \arrow[d, "\text{lit. ba.}"] \arrow[dd, bend right] \\ X' \arrow[d] \\ X'' \end{tikzcd}$$
conv

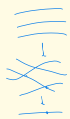
$f : X \to \mathbb{P}_1$
F = special

$$0 \to f^*\Omega^1_{\mathbb{P}_1} \to \Omega^1_X \to \Omega^1_{X/Y} \to 0$$
$$\mathcal{E} \to \mathcal{F} \hookrightarrow \Omega^1_X$$

K
$$\begin{tikzcd} \hat X \arrow[r, "\gamma"] \arrow[d] & X \arrow[d] \\ C \arrow[r] & \mathbb{P}_1 \end{tikzcd}$$

$$\begin{tikzcd} \mathrm{Norm} \arrow[d] & \\ \tilde C \times_C \tilde C \arrow[r] \arrow[d, "\delta"] & \tilde C \arrow[d, "\gamma_v"] \\ \tilde C \arrow[r, "\gamma_h"] & C \end{tikzcd}$$
μ nil

$$\gamma_h^*(\gamma_{v*}\mathcal{O}_{\tilde C}) = \delta_* \mathcal{O}_{\tilde C \times_C \tilde C} \subset \mu_* \mathcal{O}_{\mathrm{Norm}}$$



WQ $\dim X = 2$, special, $\kappa(X) = 1$
$\tilde q(X) = 0 \overset{?}{\Rightarrow} \underset{=0}{\mathrm{Alb}(X)} = \underset{\overset{!}{=}0}{\mathrm{Alb}(X, 0)} \Leftrightarrow \forall \gamma : \hat X \to X,\ h^0(\hat X, \gamma^*\Omega^1_X) = 0$.

Campana: $\exists\ X \to B \cong \mathbb{P}^1$, ~~ellipt.~~
keine mult. Fasern; ellipt. Faserung.

$\Rightarrow \exists$ nng. Faser (da nicht im Modulraum); diese sind aus rat. Kurven gebaut; diese werden von $\mathbb{C}$-Alb kontrahiert; die $\mathbb{C}$-Alb faktorisiert also über die Basis (erst einmal als Morphismus von Varietäten); Stefan: sollte auch in orbifold-Kategorie gelten $\Rightarrow \mathrm{Alb}(X, 0) = \{pt\}$.

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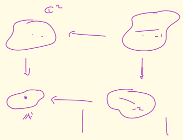
jetzt
$\dim X = 3$, glatt, special, $\tilde q(X) = 0$
Beh: $\mathrm{Alb}(X, 0) = \mathrm{Alb}(X) = \{pt\}$

1. Fall: X uniruled
$\Rightarrow$ MRC $X \xrightarrow{\mathbb{F}} Y$
↑ reproduziert Annahmen ⊕ Schritt 1
$\Rightarrow$ ok!

2. Fall: X nicht uniruled

↳ betrachte K

2.1: $K(X) = 0$
Suppose $\exists \gamma : \hat X \to X$ (branched)
$$h^0(\gamma^*\Omega^1_X) \neq \{0\}$$



$$\begin{tikzcd} X \arrow[d, equal] & & \hat{\hat X} \arrow[ll, "\hat\gamma"'] \arrow[ddd] \\ X \arrow[d] \arrow[dd, bend right] & \hat X \arrow[l, "\gamma"'] \arrow[d] & \\ X_{min} \arrow[d, dashed] & \mathrm{Torus}_1 \arrow[l, "\text{quasi-étale}"'] & \\ \mathrm{Alb}(X, 0) & & \mathrm{Torus}_2 \arrow[ll] \end{tikzcd}$$

$$\begin{tikzcd} X \arrow[d] & \hat X \arrow[l, "\gamma"'] \arrow[d, "\text{gen. endl}"] \\ X_{min} & T_3 \arrow[l] \end{tikzcd}$$
$\hat X$ ← hier ex. adpt. diff, also pluri